Strategic Environmental Policy, International Trade and Self-enforcing Agreements: The Role of Consumers' Taste for Variety

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# Strategic Environmental Policy, International Trade and Self-enforcing Agreements: The Role of Consumers' Taste for Variety

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**Abstract**

We study the coordination of environmental policy within an agreement in the context of international trade. In an n-country intra-industry trade model, firms produce a horizontally differentiated good and consumers have a taste for variety. Governments choose strategically an emission tax and their membership in an international agreement. We show that only a strong taste for variety reduces the competition among governments sufficiently enough to allow for some form of policy coordination, though full cooperation will never be obtained.

Keywords: strategic environmental policy, international trade, self-enforcing international agreements, horizontal product differentiation, taste for variety

JEL Classification: C72, F18, Q58.

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# 1 Introduction

Reaching a meaningful international agreement on climate change has proved difficult over the last three decades. The Kyoto Protocol, signed in 1998, could not even stop the trend of a continuous increase of greenhouse gas emissions world-wide observed since the last century. In the most recent round of climate change negotiations in Paris in December 2015, even though many countries around the world signed an agreement, it is only based on voluntary pledges of governments, without any enforcement mechanism in case of non-compliance. Moreover, even if all governments would deliver on their pledges, global temperature is expected to increase by 2.7-3 degrees Celsius, UNFCCC (2015), much above the widely accepted and recommended target of limiting the temperature increase by 2100 to 2 degrees Celsius compared to per-industrial levels.

Scholars in the game-theoretic literature on the formation of self-enforcing international environmental agreements (IEAs) attribute the difficulty in reaching effective climate change agreement to strong free-riding incentives. These incentives emerge because any non-signatory can enjoy the environmental benefits from reduced emissions without incurring any cost. In the absence of supranational authority that could enforce cooperation on climate change, self-enforcing agreements achieve relatively little. A central finding of this literature is that either participation in an agreement is small or if it is large, then the difference between cooperative and non-cooperative behaviour is small, i.e cooperation does not really matter. Barrett (1994a) called this the paradox of cooperation. For a recent survey of the literature, including a collection of the most influential papers over the last two decades, see Finus and Caparrós (2015).

Another body of literature explaining the slow progress in addressing transboundary pollution problems, in particular climate change, points at the fear of governments to loose competitiveness in international trade if they pursue a stricter environmental policy than other governments. Based on an extension of the simple strategic trade policy model of Brander and Spencer (1985), strategic environmental policy has been analysed for instance by Barrett (1994b), Conrad (1993) and Kennedy (1994). Under Cournot-competition[1], Brander and Spencer have shown that governments have an incentive to subsidies production of own firms in order to increase their rent capture. For environmental policy this means that emission taxes are set below marginal damages (Barrett 1994b). This result has been modified in several directions by considering additional components in governments' welfare function. Adding consumers to such a model lowers environmental taxes even further as the consumer surplus increases in the quantity produced and consumed (Kennedy 1994). Similarly, departing from the assumption of a local pollutant and considering transboundary pollution provides further incentives to lower environmental taxes below marginal

[1] Barrett (1994b) has shown that, probably not surprisingly, many of the results reverse if Bertrand-competition is considered.

damages. Some environmental damages can be externalised and governments understand that domestic production is substituted by foreign production if heavily taxed, which may even increase environmental damages if foreign is more dirty than domestic production (Conrad 1993). However, there is one effect which goes into the opposite direction (Barrett 1994b and Kennedy 1994): if there is oligopolistic competition within a country, governments have incentive to increase emission taxes in order to lower output and to establish a cartel solution. Taken together, strategic trade models offer a rich setting to explain why environmental policies may be distorted, which is also evident from Ulph (1996a and 1996b), considering also the incentive of firms to strategically invest in R&D and by allowing governments to use different environmental policy instruments.

Essentially for a long time, both strands of literature have not been integrated. That is, the IEA literature did not explicitly consider trade and the strategic environmental policy and trade literature did not allow for the formation of agreements, i.e. it did not consider the possibility that governments coordinate their policies. Only recently, Eichner and Pethig produced a series of papers considering both aspects, Eichner and Pethig (2012, 2013, 2014a and 2014b), though their trade model is very different from those mentioned above, and hence their results are difficult to relate to this literature. Overall, these series of papers seem to confirm the paradox of cooperation, with slightly more positive conclusions if the coalition behaves as Stackelberg leader. In contrast, we offer an IEA-model which is very much in the spirit of the strategic environmental policy and trade literature. Our model considers governments which care for the profits of their firms, the utility of their consumers and environmental damages, which are the result of a global pollutant. They choose strategically an emission tax and their membership in an international agreement. We allow for horizontal product differentiation where consumers' taste for variety is captured. Thus, our paper benefits from contributions by Yi (1996) and (2000) and Loke and Winters (2012) who look at international trade, trade agreements and taste for variety, but who ignore environmental damages and their effect on governments' strategic behaviour.

Our results confirm the pessimistic conclusion which mainly emerges from the IEA literature. Agreements are small at best and may not exist at all due to strong free-rider incentives. If consumers have a low taste for variety, i.e. domestic and foreign varieties are viewed as good substitutes by consumers, agreement formation fails. Only with a sufficiently high taste for variety, strategic interaction of governments is sufficiently mitigated such that small agreements are stable. In what follows, section 2 presents our model and some important properties, helpful in analysing coalition formation. Section 3 develops our results, including an in-depth analysis of the driving forces of coalition formation and the strategic interaction between signatories and non-signatories to an agreement. In section 4, we summarise our main results and conclude.

# 2 Model

## 2.1 Payoff Function

Consider an intra-industry trade model with $n$ *ex ante* symmetric countries with a representative firm and consumer in each country. We denote the set of countries by $N$. Firms produce a horizontally differentiated good, i.e. the same good but in different varieties where each firm produces one variety. Firms compete in a Cournot-fashion. Markets are segmented and each firm supplies its good to the domestic and all foreign markets. Because of the segmentation of markets, firms play a separate Cournot game in each market.[2] Transport costs are assumed away as usual.

The welfare of country $i$ is given by:

$$W_i = CS_i + PS_i + TR_i - D_i \tag{1}$$

where $CS_i$ represents country $i$'s consumer surplus, $PS_i$ country $i$'s producer surplus, $TR_i$ is the tax revenue from the emission tax imposed by the government $i$ on its domestic firm, and $D_i$ is the pollution damage faced by country $i$.

Consumers are identical and their preferences are represented by a quasi-linear utility function over two goods (see equation (2) below). The first good is the horizontally differentiated and traded good. The second good is a numeraire good, representing the composition of all other goods. Utility is linear in the numeraire good and quadratic in the differentiated good.

We assume that consumers have a taste for variety (Dixit and Stiglitz, 1977). That is, their utility depends not only on the total quantity consumed but also on the composition of quantities of the differentiated good (Yi, 1996 and 2000). The taste for variety (abbreviated TFV hereafter) is captured by parameter $\gamma \in [0,1]$. High values of $\gamma$ imply a low taste for variety and for $\gamma = 1$ varieties are perfect substitutes. In contrast, low values of $\gamma$ represent a high preference for a diverse and balanced consumption bundle and for $\gamma = 0$ varieties cannot be substituted at all.[3]

More specifically, let the representative consumer's utility in country $i$ be given by $u_i$:

$$u_i(q_i; M_i) = v_i(q_i) + M_i = aQ_{i.} - \frac{\gamma}{2}Q_{i.}^2 - \frac{1-\gamma}{2}\sum_{k \in N} q_{ik}^2 + M_i \tag{2}$$

where $v_i$ represents the utility from consuming the horizontally differentiated and traded good and $M_i$ represents the utility from consuming the numeraire good; $q_i =$

[2] See Appleyard and Field (2014) as well as Helpman and Krugman (1985) for further background.

[3] An extension could be the "ideal variety" approach where consumers have not only a general preference for the variety of a good but also a preference for a particular variety. One application is a bias towards the domestically produced variety (Di Comite *et al*, 2014).

$(q_{i1}, ..., q_{in})$ is a vector of varieties consumed by consumers in country $i$, with $q_{ik}$ representing country $i$'s consumption of country $k$'s variety[4]; $a$ is a positive demand parameter and $Q_{i.} = \sum_{k \in N} q_{ik}$ is country $i$'s total consumption of all varieties, supplied by all countries $k$.

In this paper, in most parts, we will focus our analysis on two extreme TFV scenarios for analytic tractability: the "no TFV" scenario with $\gamma = 1$ and the "maximum TFV" scenario with $\gamma = 0$.

From (2), country $i$'s inverse demand function for country $k$'s variety follows from:

$$p_{ik} = \frac{\partial u_i}{\partial q_{ik}} \Longleftrightarrow p_{ik} = a - (1-\gamma)q_{ik} - \gamma Q_{i.} \Longleftrightarrow p_{ik} = a - q_{ik} - \gamma \sum_{l \in N, l \neq k} q_{il} \qquad (3)$$

where $p_{ik}$ represents the price faced by consumers in country $i$ consuming the variety of country $k$ and $\sum_{l \in N, l \neq k} q_{il}$ is the sum of all consumed varieties produced by all firms except firm $k$ in country $k$.

From (2) and (3) , the consumer surplus in country $i$ is given by :

$$CS_i = aQ_{i.} - \frac{\gamma}{2}Q_{i.}^2 - \frac{1-\gamma}{2}\sum_{k \in N} q_{ik}^2 - \sum_{k \in N} q_{ik}p_{ik} \qquad (4)$$

where the last term in (4) represents consumers' spending.

The producer surplus of representative firm $i$ (in country $i$) is the sum of its profits in each market:

$$PS_i = \sum_{k \in N} \pi_{ki} = \sum_{k \in N} q_{ki}(p_{ki} - c - t_i) \qquad (5)$$

where $\pi_{ki}$ is firm $i$'s profit in market $k$ from selling quantity $q_{ki}$ at price $p_{ki}$ where $c$ is the constant marginal cost and $t_i$ is the emission tax imposed by country $i$'s government on its firm's output, which assumes that emissions are linked to quantities by a constant emission-output coefficient. Without loss of generality, we set this coefficient to 1.

The tax revenue, $TR_i$ , is given by:

$$TR_i = t_i \sum_{k \in N} q_{ki} \qquad (6)$$

[4] Throughout the paper the first subscript indicates the market in which the variety is consumed and the second subscript indicates the market in which it is produced.

and damages from global pollution faced by country $i$ are given by:

$$D_i = \delta \sum_{i \in N} Q_{i.} \tag{7}$$

where $\delta$ is a damage parameter, $\sum_{i \in N} Q_{i.}$ is total consumption in every country $i$ and hence total emissions (due to our assumption of a constant emission output coefficient of 1). That is, emissions constitute a pure public bad: damages depend on total emissions.

## 2.2 Coalition Formation Game

We assume a three-stage coalition formation game, which unfolds as follows.

*Stage 1, Choice of Membership:* all countries decide simultaneously whether to join coalition $S$ with $m$ the cardinality of $S$. Countries which do not join $S$ act as singletons. A typical signatory will be denoted by $i$ and a non-signatory by $j$.

Following d'Aspremont et al (1983), a coalition is called stable if it is internally and externally stable. Internal stability means that no signatory has an incentive to leave coalition $S$, whereas external stability means that no non-signatory has an incentive to join coalition $S$. We assume for simplicity that in the case of indifference a non-signatory joins coalition $S$.

Internal stability:

$$W_i(S) - W_i(S \setminus \{i\}) \geq 0 \ \forall \, i \in S \tag{8}$$

External stability:

$$W_j(S) - W_j(S \cup \{j\}) > 0 \ \forall \, j \in N \setminus S. \tag{9}$$

*Stage 2, Choice of Policy Level:* all countries choose simultaneously their emission tax.

- Signatories choose their joint emission tax $t_i$ (implemented uniformly in all signatory countries) in order to maximise the joint welfare of coalition $S$: $\underset{t_i}{max} \sum_{i \in S} W_i$
- Non-signatories choose their individual tax $t_j$ in order to maximise their individual welfare: $\underset{t_j}{max}\, W_j$ .

*Stage 3, Choice of Output:* all firms choose simultaneously and non-cooperatively their segmented market outputs by maximising profits: $\underset{q_{1i},\cdots,q_{ni}}{max} \ PS_i$ .

The game is solved by backwards induction.

## 2.3 Properties of the Game

We define the following properties to analyse the incentive structure to form coalitions and the associated welfare implications.

For all $S \subset N$, $S \neq \emptyset$, and for all $S^{'} = S \cup \{j\}$ where $S^{'} \subseteq N$:

- **Superadditivity:** a coalition game is (strictly) superadditive if:

$$\sum_{i \in S'} W_i(S^{'}) \geq (>) \sum_{i \in S} W_i(S) + W_j(S)\,.$$

- **Positive Externality:** a coalition game exhibits a (strict) positive externality if:

$$W_j(S^{'}) \geq (>) W_j(S) \;\forall\, j \notin S \, and \, j \notin S^{'}\,.$$

- **Full Cohesiveness:** a coalition game is (strictly) fully cohesive if:

$$\sum_{i \in S'} W_i(S^{'}) + \sum_{j \in N \setminus S'} W_j(S^{'}) \geq (>) \sum_{i \in S} W_i(S) + \sum_{j \in N \setminus S} W_j(S)\,.$$

Superadditivity provides an incentive to join a coalition whereas the positive externality captures the incentive to free-ride. In terms of forming large stable coalitions, the two properties work in opposite directions and typically for large coalitions the positive externality effect is stronger than the superadditivty effect. Full cohesiveness justifies the search for large stable coalitions, even if the grand coalition is not stable. Essentially, global welfare increases when the coalition is enlarged gradually and obtains its maximum in the grand coalition.

# 3 Results

## 3.1 Third Stage

In this section, we derive results for the third stage. The profit of firm $i$ in market $k$ is given by $\pi_{ki} = q_{ki}(p_{ki} - c - t_i)$. Substituting the inverse demand function from equation (3) above, we derive the following first order condition:

$$\frac{\partial \pi_{ki}}{\partial q_{ki}} = a - c - t_i - (2-\gamma) q_{ki} - \gamma Q_{k.} = 0 \iff a - c - t_i - 2q_{ki} - \gamma \sum_{l \in N, l \neq i} q_{kl} = 0 \quad (10)$$

where $Q_{k.}$ is the total quantity consumed in market $k$ and $\sum_{l\in N, l\neq i} q_{kl}$ is the sum of all consumed varieties by consumers in market $k$ from all firms except from firm $i$. It is easy to see that reaction functions ($q_{ki} = r_i(\sum_{l\in N, l\neq i} q_{kl})$ have a slope of $-\gamma/2$. Hence, the equilibrium is unique; the absolute value of the slope of the reaction function increases with the taste of variety parameter $\gamma$ and as $\gamma$ approaches zero, the strategic interaction among firms vanishes. Moreover, it is easy to see that a necessary condition for positive quantities is $a > c$. Below, we will further develop this non-negativity condition in order to ensure interior solutions.

Solving the $n$ first order conditions in market $k$ simultaneously, gives:

$$q_{ki} = \frac{(a-c)(2-\gamma) - t_i(\gamma(n-2)+2) + \gamma \sum_{k\in N, k\neq i} t_k}{(\gamma(n-1)+2)(2-\gamma)}. \tag{11}$$

Since the tax is imposed on production, the equilibrium quantity of firm $i$'s variety is the same in all markets $k$. It is evident that quantities decrease in own taxes and increase in foreign taxes. If we already account for the fact of a symmetric tax equilibrium in stage 2 with all signatories choosing the same tax rate $t_i^*$ and all non-signatories choose the same tax rate $t_j^*$ (and typically $t_i^* \neq t_j^*$), then we have for a signatory's firm

$$q^*_{.i\in S} = \frac{(a-c)(2-\gamma) - t_i^*(\gamma(n-m-1)+2) + t_j^*(\gamma(n-m))}{((n-1)\gamma+2)(2-\gamma)} \tag{12}$$

and for a non-signatory's firm

$$q^*_{.j\notin S} = \frac{(a-c)(2-\gamma) + \gamma m t_i^* - t_j^*(\gamma(m-1)+2)}{((n-1)\gamma+2)(2-\gamma)} \tag{13}$$

with the total equilibrium consumption in market $k$, $Q^*_{k.}$ given by:

$$Q^*_{k.} = \frac{n(a-c) - t_j^*(n-m) - m t_i^*}{(n-1)\gamma+2} \tag{14}$$

This leads to the following conclusions.

**Proposition 1 - The Effects of Taxes on Equilibrium Quantities**

*Consider the third stage and a market $k$. Suppose a coalition $S$ has formed in the first stage and all players have chosen their equilibrium taxes in stage 2.*

*The quantity of firm $i$'s ($j$'s) variety in a signatory country (non-signatory country) decreases with the level of signatories' (non-signatories') equilibrium taxes,* $\frac{\partial q_{ki}}{\partial t_i^*} < 0$

*($\frac{\partial q_{kj}}{\partial t_j^*} < 0$) and increases with the level of non-signatories' (signatories') equilibrium taxes, $\frac{\partial q_{ki}}{\partial t_j^*} > 0$ ($\frac{\partial q_{kj}}{\partial t_i^*} > 0$), except for $\gamma = 0$ in which case $\frac{\partial q_{ki}}{\partial t_j^*} = 0$ ($\frac{\partial q_{kj}}{\partial t_i^*} = 0$). The total quantity in market $k$ decreases in signatories' and non-signatories equilibrium tax, $\frac{\partial Q_{k.}^*}{\partial t_i^*} < 0$ and $\frac{\partial Q_{k.}^*}{\partial t_j^*} < 0$ irrespective of $\gamma$.*

**Proof:** Follows directly from equations (12) to (13) above. **Q.E.D.**

Thus, quantities produced by a firm for a particular market are negatively affected by own taxes and positively affected by foreign taxes. Given that a firm produces the same quantities for all markets, also the same holds for total production of a firm. Only for the maximum TFV, *e.g.* $\gamma = 0$, will a firm's output not be affected by the tax of a foreign government imposed on a foreign firm. Then, essentially, firms act in each segmented market like a monopolist as consumers do not substitute different varieties at all. In other words, firms do not compete and hence are only affected by their own government's taxes.

The same relationship will hold when considering second stage equilibrium taxes, with essentially two groups of players. Signatories' taxes influence non-signatories' quantities negatively and vice versa, except for $\gamma = 0$. Hence, for instance, if governments in signatory countries want to boost their firms profits by subsidizing their firms, this will automatically reduce foreign firms' quantities. However, if they decide to tax their firms to reduce total output in order to stabilize the market price, then this objective is only partially achieved because foreign firm's output will increase. A similar conflict occurs if signatories tax their firms to reduce environmental damages because foreign quantities and hence emissions will increase. Only for $\gamma = 0$ this strategic interaction breaks down.

## 3.2 Second Stage

In this section, we derive equilibrium taxes in the second stage. In order to analyse the importance of each welfare component on equilibrium taxes, we consider four welfare scenarios, with the last scenario describing our full model.

1. $W_i^1 = PS_i + TR_i$
2. $W_i^2 = CS_i + PS_i + TR_i$
3. $W_i^3 = PS_i + TR_i - D_i$
4. $W_i^4 = CS_i + PS_i + TR_i - D_i$

The first scenario replicates the simple Brander and Spencer (1985) model of trade, henceforth abbreviated as B&S-model. Consumers are ignored due to the assumption that all quantities are sold to a third market. The second and the third scenarios add one welfare component, consumer surplus and damages, respectively. The fourth scenario represents our full model. For analytic tractability, we henceforth consider two parameter values of $\gamma$, namely the "no TFV" scenario with $\gamma = 1$, and the "maximum TFV" scenario with $\gamma = 0$. Equilibrium taxes for each welfare scenario are given in Appendix 1. We denote signatories' equilibrium taxes under welfare scenario 1 by $t_i^*(PS, TR)$, scenario 2 by $t_i^*(CS, PS, TR)$ and so on, and the same applies for non-signatories' equilibrium taxes.

Inserting equilibrium taxes into equilibrium quantities reveals that we need to impose non-negativity constraints on parameter values in order to ensure positive outputs. Essentially, these constraints boil down to requesting that the demand parameter $a$ is larger than marginal production cost $c$ plus a multiple of marginal damages. The exact constraints are stated in Appendix 2, which henceforth are assumed to hold.

We now consider how signatories' and non-signatories' taxes change across the different welfare scenarios, taking welfare scenario 1, the B&S scenario, as a benchmark.

**Proposition 2 - Comparing Equilibrium Taxes Across Different Welfare Scenarios**

*Assume some coalition with $m$ signatories has formed in the first stage and let $n > m > 2$.*

*Signatories' taxes:*

- $t_i^*(PS, TR, D) > t_i^*(PS, TR) > t_i^*(CS, PS, TR)$ *for* $\gamma = \{0, 1\}$.
- $t_i^*(PS, TR, D) > t_i^*(CS, PS, TR, D) > t_i^*(CS, PS, TR)$ *for* $\gamma = \{0, 1\}$.

*Non-signatories' taxes:*

*For $\gamma = 1$:*

- $t_j^*(PS, TR, D) < t_j^*(PS, TR) < t_j^*(CS, PS, TR)$.
- $t_j^*(PS, TR, D) < t_j^*(CS, PS, TR, D) < t_j^*(CS, PS, TR)$.

*For $\gamma = 0$:*

- $t_j^*(PS, TR, D) > t_j^*(PS, TR) > t_j^*(CS, PS, TR)$.
- $t_j^*(PS, TR, D) > t_j^*(CS, PS, TR, D) > t_j^*(CS, PS, TR)$.

**Proof:** See Appendix 3. **Q.E.D.**

We first note that signatories' equilibrium taxes are lowered compared to the Brender and Spencer scenario when consumers enter governments' welfare function and are increased when instead damages are considered by governments. The reason is that the consumer surplus is negatively affected by taxes whereas damages are reduced through taxes. Hence, in terms of equilibrium taxes, consumers call for lower and damages for higher equilibrium taxes. Since both effects go in opposite directions, equilibrium taxes in the full model may be higher or lower than those in the B&S-scenario. De facto this depends on the relative weight of the consumer and damage component in the welfare function. In our model, the larger the damage parameter $\delta$ compared to the demand parameter $a$ the higher will be the tax and vice versa.

For non-signatories, we observe the same ranking as for signatories if $\gamma = 0$ because then the strategic interaction among firms vanishes. As shown in Proposition 1, if $\gamma = 0$ , quantities only depend on own taxes. In contrast for $\gamma = 1$, the strategic interaction among firms is at its maximum and hence also among governments. The ranking of equilibrium taxes for the different welfare scenarios of non-signatories is reversed to those of signatories. For instance, adding the damages to the B&S welfare scenario leads to lower equilibrium taxes for non-signatories, already indicating the strategic interaction among signatories and non-signatories, where non-signatories free-ride on signatories' emission reduction efforts. This is one version of the free-rider behavior of non-signatories undermining the formation of large stable coalitions which will be analyzed in more detail below.

We now turn to comparing signatories' and non-signatories' taxes within each welfare scenarios, which gives further insights into the strategic interaction among signatories and non-signatories.

**Proposition 3 - Comparing Equilibrium Taxes within each Welfare Scenario**

*Scenario* $W_i^1 = PS_i + TR_i$*:*

- *For* $\gamma = 1$*:* $t_i^*(PS, TR) > t_j^*(PS, TR)$*;* $\frac{\partial t_i^*(PS,TR)}{\partial m} > 0$ *and* $\frac{\partial t_j^*(PS,TR)}{\partial m} < 0$.
- *For* $\gamma = 0$*:* $t_i^*(PS, TR) = t_j^*(PS, TR)$*;* $\frac{\partial t_i^*(PS,TR)}{\partial m} = 0$ *and* $\frac{\partial t_j^*(PS,TR)}{\partial m} = 0$.

*Scenario* $W_i^2 = CS_i + PS_i + TR_i$*:*

- *For* $\gamma = 1$*:* $t_i^*(CS, PS, TR) = t_j^*(CS, PS, TR)$*;* $\frac{\partial t_i^*(CS,PS,TR)}{\partial m} = 0$ *and* $\frac{\partial t_j^*(CS,PS,TR)}{\partial m} = 0$.
- *For* $\gamma = 0$*:* $t_i^*(CS, PS, TR) < t_j^*(CS, PS, TR)$*;* $\frac{\partial t_i^*(CS,PS,TR)}{\partial m} < 0$ *and* $\frac{\partial t_j^*(CS,PS,TR)}{\partial m} = 0$.

*Scenario* $W_i^3 = PS_i + TR_i - D_i$:

- *For* $\gamma = 1$: $t_i^*(PS,TR,D) > t_j^*(PS,TR,D)$; $\frac{\partial t_i^*(PS,TR,D)}{\partial m} > 0$ *and* $\frac{\partial t_j^*(PS,TR,D)}{\partial m} < 0$.
- *For* $\gamma = 0$: $t_i^*(PS,TR,D) > t_j^*(PS,TR,D)$; $\frac{\partial t_i^*(PS,TR,D)}{\partial m} > 0$, *and* $\frac{\partial t_j^*(PS,TR,D)}{\partial m} = 0$.

*Scenario* $W_i^4 = CS_i + PS_i + TR_i - D_i$:

- *For* $\gamma = 1$: $t_i^*(CS,PS,TR,D) > t_j^*(CS,PS,TR,D)$; $\frac{\partial t_i^*(CS,PS,TR,D)}{\partial m} > 0$ *and* $\frac{\partial t_j^*(CS,PS,TR,D)}{\partial m} < 0$.
- *For* $\gamma = 0$: $t_i^*(CS,PS,TR,D) > t_j^*(CS,PS,TR,D)$ *and* $\frac{\partial t_i^*(CS,PS,TR,D)}{\partial m} > 0$ *if* $\delta m + c \leq a < 2n\delta + c$; $t_i^*(CS,PS,TR,D) \leq t_j^*(CS,PS,TR,D)$ *and* $\frac{\partial t_i^*(CS,PS,TR,D)}{\partial m} \leq 0$ *if* $a \geq 2\delta n + c$; $\frac{\partial t_j^*(CS,PS,TR,D)}{\partial m} = 0$.

*Taken together:*

| *Welfare Scenarios* | *Direction of Change* | | *Strategic Interaction* | |
|---|---|---|---|---|
| | $\gamma = 1$ | $\gamma = 0$ | $\gamma = 1$ | $\gamma = 0$ |
| $PS_i + TR_i$ | ↑,↓ | **0** , 0 | *substitutes* | *independent* [5] |
| $CS_i + PS_i + TR_i$ | **0** , 0 | ↓, 0 | *independent* | *independent* |
| $PS_i + TR_i - D_i$ | ↑,↓ | ↑, 0 | *substitutes* | *independent* |
| $CS_i + PS_i + TR_i - D_i$ | ↑,↓ | ↑↓,0 | *substitutes* | *independent* |

**Proof:** See Appendix 4. **Q.E.D.**

There are at least two interesting aspects in Proposition 3. The first aspect relates to the comparison between signatories' and non-signatories' equilibrium taxes where the former internalize externalities within their group. The second aspect relates to the strategic interaction between signatories' and non-signatories' taxes when the coalition is enlarged.

For the B&S-scenario and for the standard assumption of $\gamma = 1$, signatories impose a higher tax than non-signatories. As firms compete in a Nash-Cournot fashion, signatories' governments de facto try to enforce a cartel solution via taxes. A reduction of output leads to higher prices. Though taxes reduce firms profits, the government

[5] The first column, Direction of Change, illustrates how signatories' and non-signatories' taxes change with the coalition size. The first entry (arrow) relates to signatories' taxes and the second to non-signatories' ones. An entry of **0** for signatories means that there is no need for coordination among players. The second column illustrates the strategic interaction between signatories' and non-signatories' taxes. Intuition would suggest that in terms of forming stable coalitions, for a given welfare scenario, this will be easier if taxes are independent than if they are strategic substitutes.

collects these taxes and hence taxes are welfare neutral in this model. If the grand coalition forms, output is identical to the output of a monopolist. For $\gamma = 0$, there is no competition among firms which act like monopolists for their own variety. Hence, there is no externality across firms and hence also not among governments. In other words, there are no externalities in the B&S-model for $\gamma = 0$.

For the second scenario, adding consumers to the B&S-scenario, signatories' and non-signatories' taxes are the same for $\gamma = 1$. Adding consumers to the B&S-model calls for lower equilibrium taxes. Compared to the B&S-model, the externality is just reversed and it happens in our model that it cancels out, which implies that signatories and non-signatories have the same equilibrium tax. This is different for $\gamma = 0$. In the B&S-model, there was no externality for $\gamma = 0$ but when adding the consumer, there is a positive externality from subsidising consumption. Therefore, signatories impose lower tax than non-signatories.

For the third scenario, adding damages to the B&S-scenario, signatories choose a higher tax than non-signatories in order to internalize the negative externality stemming from emissions. Like the price externality, the emission externality stems from output and hence taxes of signatories are increased even further compared to the B&S-scenario. In the fourth scenario, the full model, effects from scenario 2 and 3 play together. For $\gamma = 1$, this means that signatories have a higher tax than non-signatories. For $\gamma = 0$, it implies that signatories' taxes can be higher or lower than non-signatories, depending on the relative importance of the consumer surplus compared to damages for signatories' internalisation strategy, which, in our model, relates to the ratio between the demand parameter $a$, production cost parameter $c$ and damage cost parameter $\delta$.

The second aspect of Proposition 3 is the strategic interaction between signatories' and non-signatories' taxes. In the case of "no TFV" with $\gamma = 1$, signatories' and non-signatories' taxes are strategic substitutes in most welfare scenarios where signatories' taxes are increasing with the coalition size $m$ whereas non-signatories' taxes are decreasing. The exception is the second scenario because externalities cancel out. In the case of "maximum TFV" with $\gamma = 0$, signatories' and non-signatories' taxes are strategically independent. Signatories' taxes are either increasing or decreasing with the coalition size depending on the externality they are internalising, except in the first welfare scenario in which they remain constant as there is no externality. In scenario 2 (3) signatories' taxes decrease (increase) with the coalition size because because of the positive externality on consumers (damages). Scenario 4 combines the effects of scenrio 2 and 3 and hence signatories' taxes decrease with coalition size if the damand parameter $a$ is sufficiently large compared to marginal production costs and global marginal damages. For all welfare scenarios with $\gamma = 0$, non-signatories' taxes do not change with the coalition size $m$ due to the independence of varieties.

### 3.3 Properties of the Coalition Game

In this section, we analyse the properties of each welfare scenario for the two scenario: "no TFV" with $\gamma = 1$ and "maximum TFV" with $\gamma = 0$. These properties have been defined in subsection 2.3 above.

**Proposition 4 - Properties of the Coalition Game**

*In the coalition game, the properties positive externality and full cohesiveness hold strictly for each of the four welfare scenarios whenever there is an externality across players. In the scenarios where there is no externality across players, these properties hold weakly.*

*For all welfare scenarios, superadditivity holds for $\gamma = 0$ and fails for $\gamma = 1$. For $\gamma = 1$ it only holds for the move from a coalition with $n-1$ signatories to the grand coalition with $n$ signatories. More specifically:*[6]

| *Welfare Scenarios* | *Positive Externality* | | *Superadditivity* | | *Full Cohesiveness* | |
|---|---|---|---|---|---|---|
| | $\gamma = 1$ | $\gamma = 0$ | $\gamma = 1$ | $\gamma = 0$ | $\gamma = 1$ | $\gamma = 0$ |
| $PS_i + TR_i$ | $+$ | 0 | $-$ | 0 | $+$ | 0 |
| $CS_i + PS_i + TR_i$ | 0 | $+$ | 0 | $+$ | 0 | $+$ |
| $PS_i + TR_i - D_i$ | $+$ | $+$ | $-$ | $+$ | $+$ | $+$ |
| $CS_i + PS_i + TR_i - D_i$ | $+$ | $+$ | $-$ | $+$ | $+$ | $+$ |

**Proof:** See Appendix 5. **Q.E.D.**

Proposition 4 confirms that the normative property of the game full cohesiveness holds. The larger are coalitions, the larger will be global welfare, which obtains its maximum in the grand coalition. It also confirms that non-signatories benefit from the enlargement of the coalition via positive externalities. This provides an incentive to free-ride. Interestingly, the incentive to join a coalition, captured by the property superadditivity, is only positive if $\gamma = 0$ but is negative if $\gamma = 1$ and whenever coalition formation would matter (i.e. full cohensiveness holds strictly). In the latter case, signatories' taxes increase with the coalition size and the reverse is true for non-signatories as shown in Proposition 3. That is, strategies are substitutes, and hence the efforts of signatories are undermined by non-signatories' reaction. This countervailing or leakage effect renders the enlargement of the coalition not successful. As recently shown in Bayramoglu, Finus and Jacques (2016), if the move from a coalition with $m-1$ to $m$ is not superadditive, then coalition with $m$ signatories cannot be internally stable. In other words, superadditivity is a necessary (though not sufficient) condition for internal stability in a positive externality game. Hence, if superadditivity fails for all $m \leq n-1$ for $\gamma = 1$, we only need to test for

[6] *Legend: "+"= holds strictly, "0"= holds weakly, and "-" generally fails.*

stability of the grand coalition. Our overall results are summarized in Proposition 5 below, which looks at the stability of coalitions in the first stage.

## 3.4 First Stage

In this section, we present the results for the first stage, i.e. the stability of coalitions.

**Proposition 5 - Coalition Stability**

*Let $m^*$ denote the size of an internally and externally stable coalition. For the four welfare scenarios, whenever there is an incentive for countries to coordinate their policy (i.e. $\frac{\partial t_i^*}{\partial m} \neq 0$), the following results are obtained:*

*For $\gamma = 1$: $m^* = 1$ and for $\gamma = 0$: $m^* = 3$.*

*More specifically:*

| *Welfare Scenarios* | *I&ES* | |
|---|---|---|
| | $\gamma = 1$ | $\gamma = 0$ |
| $PS_i + TR_i$ | $m^* = 1$ | - |
| $CS_i + PS_i + TR_i$ | - | $m^* = 3$ |
| $PS_i + TR_i - D_i$ | $m^* = 1$ | $m^* = 3$ |
| $CS_i + PS_i + TR_i - D_i$ | $m^* = 1$ | $m^* = 3$ |

**Proof:** See Appendix 6. **Q.E.D.**

Proposition 5 shows that if $\gamma = 1$ there are no stable coalitions, except in welfare scenario 2 for which coalition formation is meaningless as externality across players cancels out. This can be related to two previous results. In Proposition 3 we showed that taxes are strategic substitutes, and in Proposition 4 we showed that this implied that superadditivity generally failed.

In contrast, if $\gamma = 0$ there is stable coalition of three countries, except in scenario 1 in which coalition formation is meaningless because there is no externality across players. This can also be inferred from our previous results. In Proposition 3 we showed that taxes are strategically independent in these scenarios, and in Proposition 4 we showed that superadditivity holds. However, interestingly, despite superadditivity holds, stable coalitions are small because of the positive externality property.[7]

It is interesting that these results for both values of $\gamma$ hold irrespective of the weight consumers and damages receive in governments' s welfare function (i.e. irrespective of the welfare scenario), which stresses that they are quite robust.

[7]It is probably not surprising that for intermediate values of $\gamma$ between 0 and 1, one finds that the equilibrium coalition size lies between $m^* = 1$ and $m^* = 3$.

# 4 Concluding Remarks

In this paper, we analyzed a strategic trade model in the spirit of Brander and Spencer (1985). We introduced three additional features, which have been considered in the literature, though in isolation. Firstly, consumers matter for governments because goods are not sold to a third market. Moreover, environmental damages matter because production releases a global pollutant. Second, we consider horizontal product differentiation with consumers having a taste for variety (TFV). For analytical tractability, we focused on two extreme assumptions of TFV: no TFV and maximum TFV where the former assumption corresponds to the standard assumption in the literature that goods are perfect substitutes. Thirdly, we considered the possibility that governments can coordinate their policy by forming coalitions. Policy coordination is related to an emission tax, which is de facto an output tax because of a constant output-emission ratio. Stability of a coalition leading to an agreement was tested by invoking the concept of internally and externally stable cartels.

We demonstrated that the formation of agreements is globally beneficial. Global welfare increases with the size of agreements and obtains its maximum if the grand coalition forms (full cohesiveness). However, the grand coalition or even smaller coalitions may not be stable because of two reasons. Firstly, the benefits from policy coordination are non-exclusive, a features which we related to the property of positive externality of coalition formation. Secondly, the gains from cooperation for those involved in enlarging coalitions may be small or even negative if policy instruments are strategic substitutes. That is, superadditivity fails.

We showed that for the "no TFV" scenario, signatories of an agreement increase their taxes with the size of the agreement. Signatory governments have an incentive to internalise two negative externalities, both associated with high quantities. A reduction of output stabilises the price in the output cartel and also reduces environmental damages. Non-signatories free-ride on signatories' efforts and lower their taxes. Hence, taxes are strategic substitutes between signatories and non-signatories. In our model, this meant that no agreement was stable. In contrast, for the "maximum TFV" scenario, foreign taxes have no effect on domestic firms' output. In the context of an agreement, this implies that taxes of signatories (non-signatories) have no effect on the output of non-signatories' (signatories') firms. We found that this implies that taxes between signatories and non-signatories become strategically independent. Regardless whether signatories increase or decrease their tax with the size of the agreement, non-signatories' equilibrium taxes do not change. This reduces the free-rider incentive, but it remains positive, which explains that this led only to small stable coalitions.

To our knowledge, this is the first attempt to introduce consumers' taste for variety to the literature of international environmental agreements and trade. Our stylized model allows for exploring future research avenues in terms of additional policy instruments, like tax border adjustments, relaxing the symmetry assumption and further investigations of sub-features of TFV, such as ideal varieties or asymmetric consumers' TFV between countries.

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# 5 Appendixes

A detailed appendix with the full details of all derivations is available upon request. Below, we summarise the most important steps in the derivation in a compact form for $\gamma = 1$ and $\gamma = 0$.

## 5.1 Equilibrium Taxes for all Scenarios

For each scenario, we derive the F.O.C.s for signatories and non-signatories in stage 2 of the game. Solving these conditions simultaneously, we find the equilibrium taxes for signatories and non-signatories.

- For $\gamma = 1$:

$$t_i^*(PS, TR) = -\frac{(a-c)(n-2m+1)}{m(n(n-m+1)-m+2)}$$

$$t_j^*(PS, TR) = -\frac{(a-c)(n-1)}{n(n-m+1)-m+2}$$

$$t_i^*(CS, PS, TR) = t_j^*(CS, PS, TR) = -\frac{a-c}{n}$$

$$t_i^*(PS, TR, D) = \frac{\delta(n(m(n-m+3)-n-1)+m(2-m))-(a-c)(n-2m+1)}{m(n(n-m+1)-m+2)}$$

$$t_j^*(PS, TR, D) = -\frac{\delta(m-2)(n+1)+(a-c)(n-1)}{n(n-m+1)-m+2}$$

$$t_i^*(CS, PS, TR, D) =$$

$$\frac{n\delta(m(2-m)+n(n(m-1)+m(3-m)-1))-(a-c)(m(n^2+(1-m)(n+1))}{mn(n^2+(1-m)(n+1))}$$

$$t_j^*(CS,PS,TR,D) = -\frac{n\delta(m-2)(n+1)+(a-c)(n^2+(1-m)(n+1))}{n(n^2+(1-m)(n+1))}$$

- For $\gamma = 0$:

$$t_i^*(PS,TR) = t_j^*(PS,TR) = 0$$

$$t_i^*(CS,PS,TR) = -\frac{(a-c)m}{2n-m}$$

$$t_j^*(CS,PS,TR) = -\frac{a-c}{2n-1}$$

$$t_i^*(PS,TR,D) = \delta m$$

$$t_j^*(PS,TR,D) = \delta$$

$$t_i^*(CS,PS,TR,D) = \frac{(2n\delta-a+c)m}{2n-m}$$

$$t_j^*(CS,PS,TR,D) = \frac{2n\delta-a+c}{2n-1}$$

## 5.2 Definitions

There are certain terms that repeatedly show up in the following. They are listed below.

$$\Psi_1 = n(n-m+1)-m+2 = n^2-nm+n-m+2$$

$$\Psi_2 = n^2+(1-m)(n+1) = n^2-nm+n-m+1$$

$$\Psi_3 = n^2(m-1)-n(m-1)^2-m(m-2) = n^2m-n^2-nm^2+2nm-n-m^2+2m$$

$$\Psi_4 = n(m-1)-m(m-2) = nm-n-m^2+2m$$

$$\Psi_5 = (m-2)(n+1) = mn + m - 2n - 2$$

$$\Psi_6 = (m-1)(n+1) = mn + m - n - 1$$

$$\Psi_7 = (n-m)(n-m+1) - m(1-m) + 2 = n^2 - 2nm + 2m^2 + n - 2m + 2$$

$$\Psi_8 = n(n^2+n+1) - 2nm(n-m+1) + m^2 = n^3 - 2n^2m + 2nm^2 + n^2 - 2nm + m^2 + n$$

It can be shown that all $\Psi_k > 0$, $\forall\, n$ and $\forall\, m \leq n$.

## 5.3 Non-negativity Constraints

Inserting equilibrium taxes into equilibrium output levels, gives the quantities below, from which it is evident that for the first two welfare scenarios no non-negativity constraint needs to be imposed apart from $a > c$. For the third and fourth scenarios, additional conditions need to be imposed as explained below.

- For $\gamma = 1$:

$$q^*_{.i}(PS, TR) = \frac{(a-c)(n-m+1)}{m\Psi_1}$$

$$q^*_{.j}(PS, TR) = \frac{n(a-c)}{\Psi_1}$$

$$q^*_{.i}(CS, PS, TR) = q^*_{.j}(CS, PS, TR) = \frac{a-c}{n}$$

$$q^*_{.i}(PS, TR, D) = \frac{(a-c)(n-m+1) - \delta\Psi_3}{m\Psi_1}$$

$$q^*_{.j}(PS, TR, D) = \frac{n(a-c) + \delta(n(m-1) + m - 2)}{\Psi_1}$$

$$q^*_{.i}(CS, PS, TR, D) = \frac{(a-c)}{n} - \frac{\Psi_3\delta}{m\Psi_2}$$

$$q^*_{.j}(CS, PS, TR, D) = \frac{(a-c)}{n} + \frac{\delta n(n(m-1) + m - 2)}{n\Psi_2}$$

- For $\gamma = 0$:

$$q^*_{.i}(PS,TR) = q^*_{.j}(PS,TR) = \frac{a-c}{2}$$

$$q^*_{.i}(CS,PS,TR) = \frac{n(a-c)}{2n-m}$$

$$q^*_{.j}(CS,PS,TR) = \frac{n(a-c)}{2n-1}$$

$$q^*_{.i}(PS,TR,D) = \frac{a-c-\delta m}{2}$$

$$q^*_{.j}(PS,TR,D) = \frac{a-c-\delta}{2}$$

$$q^*_{.i}(CS,PS,TR,D) = \frac{n(a-c-\delta m)}{2n-m}$$

$$q^*_{.j}(CS,PS,TR,D) = \frac{n(a-c-\delta)}{2n-1}$$

For the third and fourth scenario, the following non-negativity constraints need to be imposed.

- For $W_i^3 = PS_i + TR_i - D_i$
  - For $\gamma = 1$: signatories' non-negativity constraint is given by $a > \tilde{a}_1 = \frac{\delta\Psi_3}{n-m+1} + c$, and for non-signatories $a > c$, with $\tilde{a}_1 > c$.
  - For $\gamma = 0$: signatories' non-negativity constraint is given by $a > \tilde{a}_2 = \delta m + c$, and for non-signatories' $a > \tilde{a}_3 = \delta + c$, with $\tilde{a}_2 > \tilde{a}_3$.
- For $W_i^4 = CS_i + PS_i + TR_i - D_i$
  - For $\gamma = 1$: signatories' non-negativity constraint is given by $a > \tilde{a}_4 = \frac{\delta n\Psi_3}{m\Psi_2} + c$, and for non-signatories by $a > c$, with $\tilde{a}_4 > \tilde{a}_2$.
  - For $\gamma = 0$: non-negativity constraints are the same as in the third scenario above ($a > \tilde{a}_2$ for signatories and $a > \tilde{a}_3$ for non-signatories).

It is is straightforward to show that $\tilde{a}_1 > \tilde{a}_4$. Throughout the whole paper, we assume the most restrictive constraint to hold for comparison within a scenario and across scenarios, noting that $n \geq m \geq 1$.

## 5.4 Proposition 2 - Comparing Equilibrium Taxes Across Different Welfare Scenarios

Assume $n \geq m > 2$ and the appropriate non-negativity constraints in section 5.3 to hold. Then, using equilibrium taxes in section 5.1, and the definitions in section 5.2, we find:

For $\gamma = 1$:

$$t_i^*(PS, TR) - t_i^*(CS, PS, TR) = \frac{(a-c)(n+1)\Psi_4}{mn\Psi_1} > 0$$

$$t_j^*(PS, TR) - t_j^*(CS, PS, TR) = -\frac{(a-c)\Psi_5}{n\Psi_1} < 0$$

$$t_i^*(PS, TR) - t_i^*(PS, TR, D) = -\frac{\delta(n+1)\Psi_4}{m\Psi_1} < 0$$

$$t_j^*(PS, TR) - t_j^*(PS, TR, D) = \frac{\delta\Psi_5}{\Psi_1} > 0$$

$$t_i^*(PS, TR, D) - t_i^*(CS, PS, TR, D) = \frac{(n+1)\Psi_4}{m\Psi_1}\left(\frac{a-c}{n} - \frac{\delta}{\Psi_2}\right) > 0$$

$$t_j^*(PS, TR, D) - t_j^*(CS, PS, TR, D) = -\frac{(m-2)(n+1)}{\Psi_1}\left(\frac{a-c}{n} - \frac{\delta}{\Psi_2}\right) < 0$$

$$t_i^*(CS, PS, TR) - t_i^*(CS, PS, TR, D) = -\frac{\delta(n+1)\Psi_4}{m\Psi_2} < 0$$

$$t_j^*(CS, PS, TR) - t_j^*(CS, PS, TR, D) = \frac{\delta\Psi_5}{\Psi_2} > 0$$

For $\gamma = 0$:

$$t_i^*(PS, TR) - t_i^*(CS, PS, TR) = \frac{(a-c)m}{2n-m} > 0$$

$$t_j^*(PS, TR) - t_j^*(CS, PS, TR) = \frac{a-c}{2n-1} > 0$$

$$t_i^*(PS, TR) - t_i^*(PS, TR, D) = -\delta m < 0$$

$$t_j^*(PS,TR) - t_j^*(PS,TR,D) = -\delta < 0$$

$$t_i^*(PS,TR,D) - t_i^*(CS,PS,TR,D) = \frac{m(a-c-\delta m)}{2n-m} > 0$$

$$t_j^*(PS,TR,D) - t_j^*(CS,PS,TR,D) = \frac{a-c-\delta}{2n-1} > 0$$

$$t_i^*(CS,PS,TR) - t_i^*(CS,PS,TR,D) = -\frac{2nm\delta}{2n-m} < 0$$

$$t_j^*(CS,PS,TR) - t_j^*(CS,PS,TR,D) = -\frac{2n\delta}{2n-1} < 0$$

## 5.5 Proposition 3 - Comparing Equilibrium Taxes within each Welfare Scenario

Using equilibrium taxes as listed in section 5.1, and the definitions in section 5.2, we find:

For $\gamma = 1$:

$$t_i^*(PS,TR) - t_j^*(PS,TR) = \frac{(a-c)\Psi_6}{m\Psi_1} > 0$$

$$t_i^*(CS,PS,TR) - t_j^*(CS,PS,TR) = 0$$

$$t_i^*(PS,TR,D) - t_j^*(PS,TR,D) = \frac{\Psi_6(n\delta + a - c)}{m\Psi_1} > 0$$

$$t_i^*(CS,PS,TR,D) - t_j^*(CS,PS,TR,C) = \frac{n\delta\Psi_6}{m\Psi_2} > 0$$

For $\gamma = 0$:

$$t_i^*(PS,TR) - t_j^*(PS,TR) = 0$$

$$t_i^*(CS,PS,TR) - t_j^*(CS,PS,TR) = -\frac{2n(a-c)(m-1)}{(2n-m)(2n-1)} < 0$$

$$t_i^*(PS,TR,D) - t_j^*(PS,TR,D) = \delta(m-1) > 0$$

$$t_i^*(CS,PS,TR,D) - t_j^*(CS,PS,TR,C) = \frac{2n(2n\delta - a + c)(m-1)}{(2n-m)(2n-1)}$$

which is positive if $\delta m + c < a \leq 2n\delta + c$ (where $\delta m + c < a$ is the non-negativity constraint) and negative if $a > 2n\delta + c$.

Furthermore:

For $\gamma = 1$:

$$\frac{\partial t_i^*(PS,TR)}{\partial m} = \frac{(n+1)(a-c)\Psi_7}{m^2\Psi_1^2} > 0$$

$$\frac{\partial t_j^*(PS,TR)}{\partial m} = -\frac{(n+1)(n-1)(a-c)}{\Psi_1^2} < 0$$

$$\frac{\partial t_i^*(CS,PS,TR)}{\partial m} = 0$$

$$\frac{\partial t_j^*(CS,PS,TR)}{\partial m} = 0$$

$$\frac{\partial t_i^*(PS,TR,D)}{\partial m} = \frac{(n+1)(n\delta+a-c)\Psi_7}{m^2\Psi_1^2} > 0$$

$$\frac{\partial t_j^*(PS,TR,D)}{\partial m} = -\frac{(n+1)(n-1)(n\delta+a-c)}{\Psi_1^2} < 0$$

$$\frac{\partial t_i^*(CS,PS,TR,D)}{\partial m} = \frac{\delta(n+1)\Psi_8}{m^2\Psi_2^2} > 0$$

$$\frac{\partial t_j^*(CS,PS,TR,D)}{\partial m} = -\frac{\delta(n+1)(n^2-n-1)}{\Psi_2^2} < 0$$

For $\gamma = 0$:

$$\frac{\partial t_i^*(PS,TR)}{\partial m} = 0$$

$$\frac{\partial t_j^*(PS,TR)}{\partial m} = 0$$

$$\frac{\partial t_i^*(CS,PS,TR)}{\partial m} = -\frac{2n(a-c)}{(2n-m)^2} < 0$$

$$\frac{\partial t_j^*(CS,PS,TR)}{\partial m} = 0$$

$$\frac{\partial t_i^*(PS,TR,D)}{\partial m} = \delta > 0$$

$$\frac{\partial t_j^*(PS,TR,D)}{\partial m} = 0$$

$$\frac{\partial t_i^*(CS,PS,TR,D)}{\partial m} = \frac{2n(2n\delta - a + c)}{(2n-m)^2}$$

which is positive if $\delta m + c < a \leq 2n\delta + c$ (where $\delta m + c < a$ is the non-negativity constraint) and negative if $a > 2n\delta + c$.

$$\frac{\partial t_j^*(CS,PS,TR,D)}{\partial m} = 0$$

## 5.6 Proposition 4 - Properties of the Coalition Game[8]

*Scenario* $W_i^1 = PS_i + TR_i$*:*

- For $\gamma = 1$:

$$EP = \frac{(2n^2 - 2nm + 3n - 2m + 5)(n+1)n^2(a-c)^2}{\Psi_1^2(n^2 - nm + 2n - m + 3)^2} > 0$$

$$SAD =$$

$$-\frac{(n^4 - 2n^3m + n^2m^2 + nm - m^2 - 4n + 3m - 1)(n+1)(a-c)^2 n}{\Psi_1^2(n^2 - nm + 2n - m + 3)^2} > 0,\ m = n,\ \&\ < 0\,\forall m < n$$

$$FC = \frac{(n^3 - 2n^2m + nm^2 + 2n^2 - 3nm + m^2 + 3n - 3m + 1)(n+1)^2(a-c)^2 n}{\Psi_1^2(n^2 - nm + 2n - m + 3)^2} > 0$$

[8]Legend: EP: Externality Property, $EP = (W_j(m)) - (W_j(m-1))$; SAD: Superadditivity, $SAD = \left(\sum_{i\in m} W_i(m)\right) - \left(\sum_{i\in(m-1)} W_i(m-1) + W_j(m-1)\right)$; FC: Full Cohesiveness $FC = \left(\sum_{i\in m} W_i(m) + \sum_{j\notin m} W_j(m)\right) - \left(\sum_{i\in(m-1)} W_i(m-1) + \sum_{j\notin(m-1)} W_j(m-1)\right)$; assuming $n \geq m > 1$.

- For $\gamma = 0$:

$$EP = 0$$

$$SAD = 0$$

$$FC = 0$$

*Scenario* $W_i^2 = CS_i + PS_i + TR_i$*:*

- For $\gamma = 1$:

$$EP = 0$$

$$SAD = 0$$

$$FC = 0$$

- For $\gamma = 0$:

$$EP = \frac{1}{2}\frac{(32n^3 + m(n(-28n + 8m) - m^2 + m - 1))(m-1)n^2(a-c)^2}{(2n-1)^2(2n-m+1)^2(2n-m)^2} > 0$$

$$SAD = \frac{1}{2}\frac{(4n^2m - 4nm^2 + m^3 + 8nm - 3m^2 - 8n + 3m)(m-1)n^2(a-c)^2}{(2n-1)^2(2n-m+1)^2(2n-m)} > 0$$

$$FC = \frac{1}{2}\frac{(32n^4 - 52n^3m + 24n^2m^2 - 3nm^3)(m-1)n^2(a-c)^2}{(2n-1)^2(2n-m+1)^2(2n-m)^2}$$

$$+\frac{1}{2}\frac{(16n^2m - 13nm^2 + 2m^3 - 16n^2 + 13nm - 2m^2)(m-1)n^2(a-c)^2}{(2n-1)^2(2n-m+1)^2(2n-m)^2} > 0$$

*Scenario* $W_i^3 = PS_i + TR_i - D_i$:

- For $\gamma = 1$:

$$EP = \frac{(n\delta + a - c)^2(2n^2 - 2nm + 3n - 2m + 5)(n+1)n^2}{\Psi_1^2(n^2 - nm + 2n - m + 3)^2} > 0$$

$$SAD =$$

$$-\frac{(n\delta + a - c)^2(n^4 - 2n^3m + n^2m^2 + nm)(n+1)n}{\Psi_1^2(n^2 - nm + 2n - m + 3)^2}$$

$$-\frac{(n\delta + a - c)^2(-m^2 - 4n + 3m - 1)(n+1)n}{\Psi_1^2(n^2 - nm + 2n - m + 3)^2} > 0,\ m = n,\ \&\ < 0\,\forall m < n$$

$$FC = \frac{(n\delta + a - c)^2(n^3 - 2n^2m + nm^2 + 2n^2 - 3nm + m^2 + 3n - 3m + 1)(n+1)^2n}{\Psi_1^2(n^2 - nm + 2n - m + 3)^2} > 0$$

- For $\gamma = 0$:

$$EP = n(m-1)\delta^2 > 0$$

$$SAD = \frac{1}{4}n(m-1)m\delta^2 > 0$$

$$FC = \frac{1}{4}n(m-1)(4n - 3m)\delta^2 > 0$$

*Scenario* $W_i^4 = CS_i + PS_i + TR_i - D_i$:

- For $\gamma = 1$:

$$EP = \frac{1}{2}\frac{\delta^2(2n^2 - 1)(2n^2 - 2nm + 3n - 2m + 3)(n+1)n^2}{(n^2 - \Psi_5)^2\Psi_2^2} > 0$$

$$SAD = -\frac{1}{2}\frac{\delta^2(2n^5 - 4n^4m + 2n^3m^2 - 4n^3 + 8n^2m)(n+1)n^2}{(n^2 - \Psi_5)^2\Psi_2^2} -$$

$$\frac{1}{2}\frac{\delta^2(-4nm^2 - 8n^2 + 9nm - 2m^2 - 4n + 3m)(n+1)n^2}{(n^2 - \Psi_5)^2\Psi_2^2} > 0,\ m = n,\ \&\ < 0, \forall m < n$$

$$FC = \frac{1}{2}\frac{\delta^2(2n^3 - 4n^2m + 2nm^2 + 4n^2 - 6nm + 2m^2 + 4n - 4m + 1)(n+1)^2n^3}{(n^2 - \Psi_5)^2\Psi_2^2} > 0$$

- For $\gamma = 0$:

$$EP = \frac{1}{2}\frac{(n\delta - a + c)^2(32n^3 - 28n^2m + 8nm^2 - m^3 + m^2 - m)(m-1)n^2}{(2n-1)^2(2n-m+1)^2(2n-m)^2} > 0$$

$$SAD = \frac{1}{2}\frac{(n\delta - a + c)^2(4n^2m - 4nm^2 + m^3 + 8nm - 3m^2 - 8n + 3m)(m-1)n^2}{(2n-1)^2(2n-m+1)^2(2n-m)^2} > 0$$

$$FC =$$

$$\frac{1}{2}\frac{(n\delta - a + c)^2(32n^4 - 52n^3m + 24n^2m^2 - 3nm^3)(m-1)n^2}{(2n-1)^2(2n-m+1)^2(2n-m)^2} +$$

$$\frac{1}{2}\frac{(n\delta - a + c)^2(16n^2m - 13nm^2 + 2m^3 - 16n^2 + 13nm - 2m^2)(m-1)n^2}{(2n-1)^2(2n-m+1)^2(2n-m)^2} > 0$$

### 5.7 Proposition 5 - Coalition Stability

*Scenario* $W_i^1 = PS_i + TR_i$:

- For $\gamma = 1$

$$W_i(S) - W_j(S\backslash\{i\}) = -\frac{n(a-c)^2(n+1)(n^4m - 2n^3m^2 + n^2m^3 - n^4 + 4n^3m - 5n^2m^2 + 2nm^3)}{m\Psi_1^2(n^2 - nm + 2n - m + 3)^2} -$$

$$\frac{n(a-c)^2(n+1)(-4n^3 + 13n^2m - 10nm^2 + m^3 - 10n^2 + 17nm - 7m^2 - 12n + 15m - 9)}{m\Psi_1^2(n^2 - nm + 2n - m + 3)^2} < 0$$

- For $\gamma = 0$:

$$W_i(S) - W_i(S \setminus \{i\}) = 0$$

*Scenario* $W_i^2 = CS_i + PS_i + TR_i$:

- For $\gamma = 1$:

$$W_i(S) - W_i(S \setminus \{i\}) = 0$$

- For $\gamma = 0$:

$$W_i(S) - W_j(S \setminus \{i\}) =$$

$$-\frac{1}{2}\frac{n^2(a-c)^2(m-1)(2nm-m^2-6n+3m-1)}{(2n-1)(2n-m+1)^2(2n-m)} > 0, \forall m \leq 3,\ \&\ < 0, \forall m > 3$$

*Scenario* $W_i^3 = PS_i + TR_i - D_i$:

- For $\gamma = 1$

$$W_i(S) - W_j(S \setminus \{i\}) = -\frac{n(n\delta+a-c)^2(n+1)(n^4m-2n^3m^2+n^2m^3-n^4+4n^3m)}{m\Psi_1^2(n^2-nm+2n-m+3)^2} -$$

$$\frac{n(n\delta+a-c)^2(n+1)(-5n^2m^2+2nm^3-4n^3+13n^2m-10nm^2)}{m\Psi_1^2(n^2-nm+2n-m+3)^2} -$$

$$\frac{n(n\delta+a-c)^2(n+1)(m^3-10n^2+17nm-7m^2-12n+15m-9)}{m\Psi_1^2(n^2-nm+2n-m+3)^2} < 0$$

- For $\gamma = 0$:

$$W_i(S) - W_i(S \setminus \{i\}) = -\frac{1}{4}n\delta^2(m-1)(m-3) \geq 0, \forall\, m \leq 3,\ \&\ < 0,\ \forall m > 3$$

*Scenario* $W_i^4 = CS_i + PS_i + TR_i - D_i$:

- For $\gamma = 1$:

$$W_i(S) - W_j(S \setminus \{i\}) = -\frac{1}{2}\frac{\delta^2 n^2(n+1)(2n^5m-4n^4m^2+2n^3m^3-2n^5+8n^4m-10n^3m^2)}{m\Psi_2^2(n^2-\Psi_5)^2} -$$

$$\frac{1}{2}\frac{\delta^2 n^2(n+1)(4n^2m^3-8n^4+22n^3m-16n^2m^2+2nm^3-16n^3)}{m\Psi_2^2(n^2-\Psi_5)^2} -$$

$$\frac{1}{2}\frac{\delta^2 n^2(n+1)(28n^2m-12nm^2-16n^2+19nm-2m^2-8n+3m)}{m\Psi_2^2(n^2-\Psi_5)^2} < 0$$

- For $\gamma = 0$:

$$W_i(S) - W_j(S \setminus \{i\}) =$$

$$-\frac{1}{2}\frac{n^2(2n\delta-a+c)^2(m-1)(2nm-m^2-6n+3m-1)}{(2n-1)(2n-m+1)^2(2n-m)} > 0, \forall m \leq 3,\ \&\ < 0, \forall m > 3$$